

# UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education Ordinary Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

# 4 6 2 3 3 1 7 7 0 8

### **ADDITIONAL MATHEMATICS**

4037/22

Paper 2

October/November 2011

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

#### **READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

#### Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

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Total	

This document consists of 16 printed pages.



# Mathematical Formulae

#### 1. ALGEBRA

Quadratic Equation

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where *n* is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!}$ .

# 2. TRIGONOMETRY

*Identities* 

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\csc^2 A = 1 + \cot^2 A$$

Formulae for  $\triangle ABC$ 

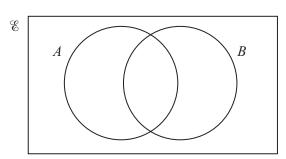
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^{2} = b^{2} + c^{2} - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

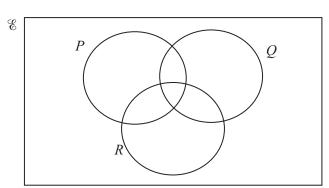
1 (a) The universal set  $\mathscr E$  and the sets A and B shown in the Venn diagram below are such that n(A) = 15, n(B) = 20,  $n(A' \cap B) = 6$  and  $n(\mathscr E) = 30$ .

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In the Venn diagram below insert the number of elements in the set represented by each of the four regions. [4

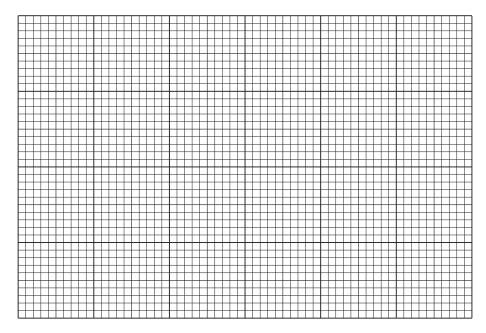


**(b)** In the Venn diagram below shade the region that represents  $(P \cup Q) \cap R'$ . [1]



2 (i) On the grid below, draw on the same axes, for  $0^{\circ} \le x \le 180^{\circ}$ , the graphs of  $y = \sin x$  and  $y = 1 + \cos 2x$ .





- (ii) State the number of roots of the equation  $\sin x = 1 + \cos 2x$  for  $0^{\circ} \le x \le 180^{\circ}$ . [1]
- (iii) Without extending your graphs state the number of roots of the equation  $\sin x = 1 + \cos 2x$  for  $0^{\circ} \le x \le 360^{\circ}$ .

It is given that 2x - 1 is a factor of the expression  $4x^3 + ax^2 - 11x + b$  and that the remainder when the expression is divided by x + 2 is 25. Find the remainder when the expression is divided by x + 1.

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4 It is given that 
$$\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 1 & -5 \end{pmatrix}$$
,  $\mathbf{B} = \begin{pmatrix} 1 & -4 & 2 \\ -3 & 5 & 0 \end{pmatrix}$  and  $\mathbf{C} = \begin{pmatrix} 4 \\ 2 \\ -7 \end{pmatrix}$ .

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(i) Calculate AB.

[2]

(ii) Calculate BC.

[2]

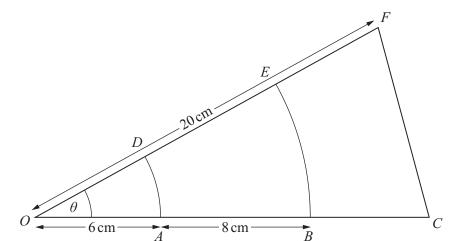
(iii) Find the inverse matrix,  $A^{-1}$ .

[2]

•	Fou this	ir boys and three girls are to be seated in a row. Calculate the number of different ways the can be done if	ıat	For Examiner's Use
	(i)	the boys and girls sit alternately,	[2]	
	(ii)	the boys sit together and the girls sit together,	[2]	
	(iii)	a boy sits at each end of the row.	[2]	

6	The length of a rectangular garden is $x$ m and the width of the garden is 10 m less than the length.									
	(i)	Given that the perimeter of the garden is greater than $140\mathrm{m}$ , write down a linear inequality in $x$ .	Examiner's							
	(ii)	Given that the area of the garden is less than $3000\mathrm{m}^2$ , write down a quadratic inequality in $x$ . [1]								
	(iii)	By solving these two inequalities, find the set of possible values of $x$ . [4]								

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In the diagram AD and BE are arcs of concentric circles centre O, where OA = 6 cm and AB = 8 cm. The area of the region ABED is 32 cm<sup>2</sup>. The triangle OCF is isosceles with OC = OF = 20 cm.

(i) Find the angle  $\theta$  in radians.

[3]

(ii) Find the perimeter of the region *BCFE*.

[5]

8	A particle travels in a straight line so that, $t$ s after passing through a fixed point $O$ , its velocity,
	$v \mathrm{ms}^{-1}$ , is given by $v = 12 \mathrm{cos} \left(\frac{t}{3}\right)$ .

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(i) Find the value of t when the velocity of the particle first equals  $2 \,\mathrm{ms}^{-1}$ . [2]

(ii) Find the acceleration of the particle when t = 3. [3]

(iii) Find the distance of the particle from O when it first comes to instantaneous rest. [4]

- 9 It is given that  $f(x) = 2x^2 12x + 10$ .
  - (i) Find the value of a, of b and of c for which  $f(x) = a(x+b)^2 + c$ .

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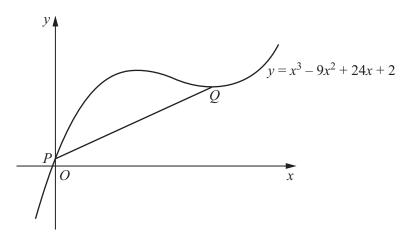
[3]

[4]

(ii) Sketch the graph of y = |f(x)| for  $-1 \le x \le 7$ .

(iii) Find the set of values of k for which the equation |f(x)| = k has 4 distinct roots. [2]

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The diagram shows part of the curve  $y = x^3 - 9x^2 + 24x + 2$  cutting the y-axis at the point P. The curve has a minimum point at Q.

(i) Find the coordinates of the point Q.

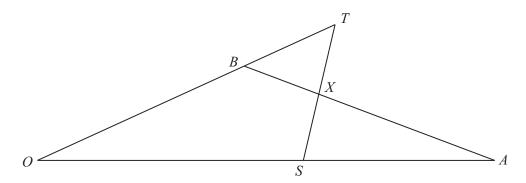
[4]

(ii)	Find the area of the region enclosed by the curve and the line $PQ$ .	[6]	For Examiner's Use

11 Answer only **one** of the following two alternatives.

**EITHER** 

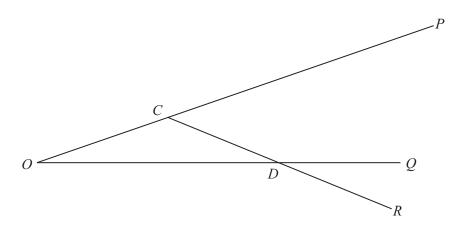
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In the diagram above  $\overrightarrow{OA} = \mathbf{a}$ ,  $\overrightarrow{OB} = \mathbf{b}$ ,  $\overrightarrow{OS} = \frac{3}{5} \overrightarrow{OA}$  and  $\overrightarrow{OT} = \frac{7}{5} \overrightarrow{OB}$ .

- (i) Given that  $\overrightarrow{AX} = \mu \overrightarrow{AB}$ , where  $\mu$  is a constant, express  $\overrightarrow{OX}$  in terms of  $\mu$ , **a** and **b**. [2]
- (ii) Given that  $\overrightarrow{SX} = \lambda \overrightarrow{ST}$ , where  $\lambda$  is a constant, express  $\overrightarrow{OX}$  in terms of  $\lambda$ , **a** and **b**. [4]
- (iii) Hence evaluate  $\mu$  and  $\lambda$ . [4]

OR



In the diagram above  $\overrightarrow{OC} = \mathbf{c}$  and  $\overrightarrow{OD} = \mathbf{d}$ . The points P and Q lie on OC and OD produced respectively, so that OC : CP = 1 : 2 and OD : DQ = 2 : 1. The line CD is extended to R so that CD = DR.

- (i) Find, in terms of **c** and/or **d**, the vectors  $\overrightarrow{OP}$ ,  $\overrightarrow{OQ}$  and  $\overrightarrow{OR}$ . [5]
- (ii) Show that the points P, Q and R are collinear and find the ratio PQ : QR. [5]

Start your answer to Question 11 here.							
Indicate which question you are answering.	EITHER					Examiner's Use	
	OR						
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Continue your answer here if necessary.	For
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